Prevalence of nowhere analytic functions

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If a function f is infinitely differentiable on an open neighbourhood of a point x_0, its Taylor series at x_0 is denoted by

$$T(f,x_0)(x) = \sum_{n=0}^{+\infty} \frac{f^{(n)}(x_0)}{n!} \, (x-x_0)^n.$$

We say that f is *analytic at* x_0 if $T(f,x_0)$ converges to f on an open neighbourhood of x_0. If this is not the case, we say that f has a singularity at x_0. A function with a singularity at each point of an interval is called *nowhere analytic* on the interval.

Several examples of infinitely differentiable nowhere analytic functions exist and generic results have already been obtained ([2, 3, 5, 6, 7]). This question of genericity can also be treated using the concept of prevalence, introduced by Hunt, Sauer and Yorke ([4]). In the talk, we show that the set of nowhere analytic functions is prevalent in $C^\infty([0,1])$. More precise results are obtained using Gevrey classes ([1]). This study is then extended to the classes of quasi-analytic functions.

References

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