Prevalence of nowhere analytic functions
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If a function $f$ is infinitely differentiable on an open neighbourhood of a point $x_0$, its Taylor series at $x_0$ is denoted by

$$T(f, x_0)(x) = \sum_{n=0}^{+\infty} \frac{f^{(n)}(x_0)}{n!} (x - x_0)^n.$$  

We say that $f$ is analytic at $x_0$ if $T(f, x_0)$ converges to $f$ on an open neighbourhood of $x_0$. If this is not the case, we say that $f$ has a singularity at $x_0$. A function with a singularity at each point of an interval is called nowhere analytic on the interval.

Several examples of infinitely differentiable nowhere analytic functions exist and generic results have already been obtained ([2, 3, 5, 6, 7]). This question of genericity can also be treated using the concept of prevalence, introduced by Hunt, Sauer and Yorke ([4]). In the talk, we show that the set of nowhere analytic functions is prevalent in $C^\infty([0,1])$. More precise results are obtained using Gevrey classes ([1]). This study is then extended to the classes of quasi-analytic functions.

References


