A PRIMER ON CLARK MEASURES

WILLIAM T. ROSS

In this series of lectures I will give a gentle introduction to the topic of Clark measures which turn out to have an uncanny way of appearing in many different guises in analysis. In short, for a bounded analytic function Θ on the open unit disk for which $\Theta(e^{i\theta})$ is unimodular for almost every θ (such functions are called inner), there is a family $\{\mu_\alpha : |\alpha| = 1\}$ of measures on the unit circle associated with Θ by the formula

$$\frac{1 - |\Theta(z)|^2}{|\alpha - \Theta(z)|^2} = \int \frac{1 - |z|^2}{|e^{i\theta} - z|^2} d\mu_\alpha(e^{i\theta}).$$

These measures, called Clark measures, appear as the spectral representing measures for a certain important unitary operator (the rank-one unitary perturbation of the compressed shift) (Clark's theorem). They disintegrate Lebeague measure on the circle (Aleksandrov's theorem). They help us understand Carleson measures for model spaces. This list goes on and on. We will cover all of these theorems along with some recent work which generalizes the Clark theory (perturbations of compressed shifts, Aleksandrov's theorem, etc.) for matrix-valued inner functions.

This course is intended for graduate students with just the basics of real analysis (measure theory, Lebesgue theory), functional analysis (Riesz representation theorem, Hahn-Banach theorem, spectral theorem), complex analysis, and of course, the basics of linear algebra. I will build everything from the ground up.

Department of Mathematics and Computer Science, University of Richmond, Richmond, Virginia, 23173, USA

E-mail address: wross@richmond.edu

URL: http://facultystaff.richmond.edu/~wross