Kreiss resolvent conditions for power bounded matrices

Rachid Zarouf

Let $T\,:(\mathbb{C}^n,\,|\cdot|)\mapsto(\mathbb{C}^n,\,|\cdot|)$ be an operator acting on a finite dimensional Banach space. We suppose that T satisfies the following *power boundedness condition*:

$$P(T)=\sup_{k\geq0}\left\Vert T^k\right\Vert_{E\to E}<\infty\,. \tag{PBC}$$

We denote by $\sigma(T)=\{\lambda_1,\,\dots,\,\lambda_n\}$ the spectrum of T, $r(T)=\max_i|\lambda_i|$ its spectral radius (which satisfies $r(T)\leq1$ since $P(T)<\infty$), and $R(z,\,T)=(zId-T)^{-1}$ the resolvent of T at point z, for $z\in\mathbb{C}\setminus\sigma(T)$, Id being the identity operator. Our problem here is to "study" the quantity $\left\Vert R(z,\,T)\right\Vert$.

The classical KRC is satisfied if and only if (by definition)

$$\rho(T)=\sup_{|z|>1}\left(|z|-1\right)\left\Vert R(z,\,T)\right\Vert<\infty. \tag{KRC}$$

The conditions (PBC) and (KRC) are equivalent: this is the well-known *Kreiss Matrix Theorem*.

Here we define for example the quantities

$$\rho_\alpha(T)=\sup_{|z|>1}\left(|z|-1\right)^\alpha\left\Vert R(z,\,T)\right\Vert,$$

where $\alpha\in(0,\,1)$, or

$$\rho^{strong}(T)=\sup_{|z|\geq1}\mathrm{dist}\left(z,\,\sigma(T)\right)\left\Vert R(z,\,T)\right\Vert,$$

and find upper estimates for these quantities in terms of $P(T)$ and in terms of datas on the spectrum $\sigma(T)$ of T. Central tools for the proofs of our results are BTIRF: *Bernstein-type inequalities for rational functions.*