

Composition operators on the half-plane

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A composition operator on a space of functions is the operator induced by some pre-composition by a self-map of the domain. In the classical Hardy and Bergman spaces of the disc, it is well known that all analytically-induced composition operators are bounded, and descriptions of the compact and isometric operators, as well as those which are similar to an isometry, are also well known. By contrast, the equivalent Hardy spaces of the half-plane, which are isometrically isomorphic to their disc counterparts, admit a variety of unbounded composition operators. Moreover, Valentin Matache showed that there are no compact composition operators on any of the Hardy spaces of the half-plane. In this talk, we discuss results by the speaker and Michael Jury (for the Hardy space case) and the speaker and Andrew Wynn (for the Bergman space case), which give necessary and sufficient conditions for the boundedness of half-plane composition operators, as well as formulations of the norm, essential norm and spectral radius of such operators. In particular, the result provides a new proof of Matache's result on compactness, and quantifies another result of his on the boundedness of such operators. If there is time, we may also discuss the question of similarity to an isometry, and its relation to the theory of universal operators and invariant subspaces.